Homework 9 (MATH 2310-04) **Name (Print):**

Due date: April 19, 2012

1. Use the Laplace transform to solve the given initial value problem.

$$y''+y'+\frac{5}{4}y = f(t), \quad y(0) = 0, \quad y'(0) = 0, \quad f(t) = \begin{cases} \sin(t) & 0 \le t < \pi \\ 0 & t \ge \pi \end{cases}$$

Solution :

$y(t) = h(t) + u_{\pi}\, h(t - \pi)$

$h(t) = (4/17)\,[-4\cos(t) + \sin(t) + 4\, e^{-t/2}\cos(t) + e^{-t/2}\sin(t)]$

2. Use the Laplace transform to solve the given initial value problem.

$$y''+2y'+2y = h(t), \quad y(0) = 0, \quad y'(0) = 1, \quad h(t) = \begin{cases} 1 & \pi \le t < 2\pi \\ 0 & 0 \le t < \pi \quad \text{and} \quad t \ge 2\pi \end{cases}$$

Solution :

$y(t) = e^{-t}\sin(t) + 0.5\, u_{\pi}\,[1 + e^{-(t-\pi)}\cos(t) + e^{-(t-\pi)}\sin(t)]$

$\quad - 0.5\, u_{2\pi}\,[1 - e^{-(t-2\pi)}\cos(t) - e^{-(t-2\pi)}\sin(t)]$

3. Use the Laplace transform to solve the given initial value problem.

$$y''+4y = \sin(t) - u_{2\pi}\sin(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 0$$

Solution :

$y(t) = [1 - u_{2\pi}]\,(2\sin(t) - \sin(2t)) / 6$