Toward Reliable Predictions of Real World Turbulent Flows

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1. The Problem

Most real flows are very turbulent: they are characterized by a very high Reynolds number (Re). If obstacles are involved, we often find separated turbulent flow as illustrated in Fig. 1. Such instantaneous, fluctuating flow regimes need to be considered to enable accurate flow predictions, e.g. to accurately predict the performance of wind farms and effectiveness of aircraft designs. However, existing predictive methods, direct numerical simulation (DNS), large eddy simulation (LES), and experiments, are hardly applicable to such high Re wall-bounded turbulent flows. Computationally more efficient Reynolds-Averaged Navier-Stokes (RANS) methods require, basically, evidence for all predictions.

Fig 1. Illustration of an airplane vortex (from Wikimedia Commons). Corresponding unsteady, separated flow regimes are found with regard to many highly essential problems, e.g. regarding the assessment of the performance of wind farms and effectiveness of aircraft designs.

2. The Core Problem: Missing Mathematics

The most reasonable approach to tackle this challenge is the use of hybrid methods (the combination of different simulations methods). Such simulations involve two ingredients: modeled motions represented by the model [the partial differential equation (PDE)] applied and produced fluctuating resolved motions if the grid is sufficiently fine. Both components, modeled and resolved motions, need to be in balance. This means the relative model contribution should be small (large) if the computational grid supports the simulation of a lot of (a little) resolved motion, see the illustration in Fig. 2. The basic problem is to enable the communication between the PDE and its produced resolved motion, and to determine an appropriate model response: it needs the introduction of stable, self-controlling PDEs.

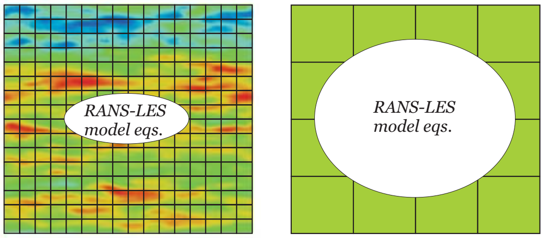

Fig 2. Required model contributions (ellipses) depending on the flow resolution (represented by fluctuations) implied by a grid (horizontal and vertical black lines). The fine grid on the left enables the simulation of resolved motion, the coarse grid on the right does not. Hence, the model contribution on the left (right) needs to be small (large).

3. Three Decades of Search for a Solution

The search for solutions to this problem continues for three decades: see Fig. 3. More than $9K$ related articles are currently published. This number can be expected to increase to $25K$ papers in 2030 [1].

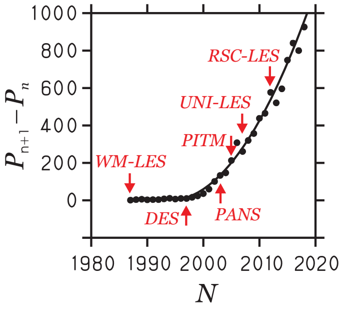

Fig 3. The number of new articles on hybrid methods per year $P_{n+1}-P_n$ at year N. The black line shows $P_{n+1}-P_n = 1.66(n+1)^2$, where $n = N-1995$. The red acronyms refer to the introduction of some hybrid methods. The cumulative number of papers is given by $P_n = 48+1.66n(n+1)(2n+1)/6$.

4. New Mathematics: Continuous Eddy Simulation (CES)

A solution to the problems described above requires, first, the introduction of corresponding self-controlling PDEs (having the ability to appropriately adjust the model contribution to the actual amount of resolved motion), and second, the demonstration that this mode redistribution mechanism stably works for wide grid and Re variations up to the asymptotic regime of extreme Re. A closely related question is whether it is possible to implement this mechanism in a variety of simulations methods. In a sequence of articles, it was recently shown how it is possible to solve these questions:

- Heinz, Prog. Aerosp. Sci., 2020 [1]
- Heinz, Phys. Fluids, 2019 [2]
- Heinz, Mokhtarpoor, Stoellinger, Phys. Fluids, 2020 [3]
- Heinz, Phys. Fluids, 2021 [4]
- Heinz, Appl. Math. Model., 2021 [5]

5. Basic Applications

CES applications are reported so far for periodic hill flows for a wide Re range up to $Re = 500K$ [3]: see the illustration in Fig. 4. No resolving LES or experiments are applicable to this flow. CES methods were found to perform very well. In particular, the following conclusions were obtained [3]:

- In contrast to other methods, the mode variation mechanism works for wide grid/Re variations.
- The mechanism can be set-up in several models.
- Predictions agree very well with validation data.
- CES predictions at extreme Re can be partially validated by the grid/Re analogy.

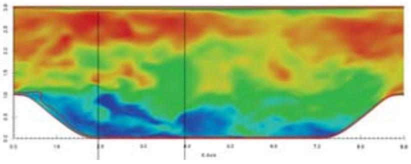

Fig 4. An illustration of periodic hill flow simulations (channel flow with enclosed hills): streamwise positive (red) and negative (blue) velocity fluctuations are shown. A recirculation zone (in blue) can be observed after the hill on the left-hand side.

6. Needed Further Developments

Further developments needed to enable routine CES predictions of turbulent flows are pictured in Fig. 5.

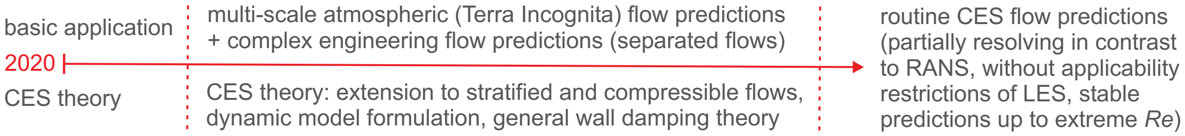

Fig 5. Needed further developments toward routine reliable predictions of real world turbulent flows.

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References

[1] S. Heinz. A review of hybrid RANS-LES methods for turbulent flows: Concepts and applications. *Prog. Aerosp. Sci.*, 114:100597/1–100597/25, 2020.

[2] S. Heinz. The large eddy simulation capability of Reynolds-Averaged Navier-Stokes equations: Analytical results. *Phys. Fluids*, 31(2):021702/1–021702/6, 2019.

[3] S. Heinz, R. Mokhtarpoor, and M. Stoellinger. Theory-based Reynolds-Averaged Navier-Stokes equations with Large Eddy Simulation capability for separated turbulent flow simulations. *Phys. Fluids*, 32 (6):065102/1–065102/20, 2020.

[4] S. Heinz. The continuous eddy simulation capability of velocity and scalar probability density function equations for turbulent flows. *Phys. Fluids*, 33(6):025107/1–025107/13, 2021.

[5] S. Heinz. Theory-based mesoscale to microscale coupling for wind energy applications. *Appl. Math. Model.*, 98:563–575, 2021.