Homework 2 (MATH 5320-01) **Name (Print):**

Due date: Thursday, Feb. 26, 2009

1. Some diseases are spread largely by carriers, individuals who can transmit the disease but who exhibit no overt symptoms. Let x and y, respectively, denote the proportion of susceptibles and carriers in the population. Suppose that carriers are identified and removed from the population at a rate β, so

$$\frac{dy}{dt} = -\beta y. \qquad (1)$$

Suppose also that the disease spreads at a rate proportional to the product of x and y,

$$\frac{dx}{dt} = -\alpha xy. \qquad (2)$$

a) Determine y at any time t by solving equation (1) subject to the initial condition $y(0) = y_0$.

b) Use the result of part a) to find x at any time t by solving equation (2) subject to the initial condition $x(0) = x_0$.

c) Find the proportion of the population that escapes the epidemic by finding the limiting value of x as $t \to \infty$.

2. Consider the following predator-prey (Lotka-Volterra) equations:

$$\frac{dx}{dt} = x(1.5 - 0.5y),$$

$$\frac{dy}{dt} = y(-0.5 + x).$$

a) Find the equilibrium (or critical) points.

b) For each equilibrium point find the corresponding linear system.

c) Find the solution of each linear system.

d) Determine the limiting behavior of x and y as $t \to \infty$.

e) Explain the results in terms of the populations of the two species.