

Homework 3 (MATH 5320-01)**Name (Print):****Due date: Tuesday, March 31, 2009**

Consider the following stochastic population model:

$$\frac{P_{n+1} - P_n}{\Delta t} = C P_n - P_n^2 + \sigma P_n \frac{\Delta W_n}{\Delta t}.$$

Here, $C = 1$, σ is any deterministic parameter, and ΔW_n is a Gaussian process with the properties

$$\langle \Delta W_n \rangle = 0,$$

$$\langle \Delta W_n \Delta W_m \rangle = \begin{cases} 0 & \text{if } m \neq n \\ \Delta t & \text{if } m = n \end{cases}.$$

- a) Assume that $\sigma = 0.2$ and $\Delta t = 0.02$. Illustrate the evolution of three realizations of P_n .
- b) Consider the stationary probability distribution (also for the following questions).
Calculate the first four moments of the probability density function (PDF) of P_n .
- c) Calculate the skewness and kurtosis based on your previous calculation of moments.
Do these values provide support for the view that the PDF of P_n is a Gaussian PDF?
- d) Compare the mean and variance of P_n with the corresponding analytical results derived in class. Calculate and discuss the error of the analytical expressions.
- e) Use the analytical expressions for the mean and variance to illustrate the variation of these variables with σ by a plot. Explain why these variations with σ are plausible.