

**Homework 4 (MATH 5320-01)**  
**Due date: Tuesday, April 21, 2009**

**Name (Print):**

Consider the interaction-by-exchange-with-the-mean (IEM) model for the turbulent mixing of scalars.

$$\frac{d\phi^*}{ds} = -(\phi^* - \bar{\phi}). \quad (1)$$

Here,  $\phi^*$  is the mass fraction of any substance (which is bounded,  $0 \leq \phi^* \leq 1$ ),  $\bar{\phi}$  is the constant mean value of  $\phi^*$ , and  $s$  represents a normalized time.

1. Under which condition represents the IEM model a stochastic model?
2. Derive and solve formally the transport equation for the probability density function of  $\phi^*$ . Interpret your results.
3. Derive and solve the equations for the mean and variance of  $\phi^*$  which are implied by the IEM model. Interpret your results.
4. Use equation (1) to derive a stochastic equation for the standardized scalar (this means the fluctuation of  $\phi^*$  divided by the square root of the variance of  $\phi^*$ ). Interpret your results.
5. Derive the corresponding transport equation for the probability density function of standardized scalars. Solve this equation and interpret your results.