**Mathematical Foundations of Future Turbulent Flow Simulations:**

**Project Outcomes Report**

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Improved understanding of optimal design conditions for a variety of turbulent flow problems would offer tremendous advantages: e.g., we would be able to optimally place wind turbines in wind farms and to reduce the cost of aircraft flight by optimal aircraft designs. The only way that enables a comprehensive assessment of such turbulent flow scenarios is the simulation of such turbulent flows based on numerically solving rather complicated partial differential equations (PDE). The requirement for applying this approach is knowledge of appropriate PDE which can be used to properly determine the spatial distribution and the development of turbulent flows.

The search for such appropriate PDE is a slow and resource-intensive process which continues now for more than fifty years. There are two mainstream approaches which are applied over a long time. One approach is based on the idea to explain the mechanism of turbulent flows on the basis of model assumptions. The other approach is based on the idea to basically resolve the flow, this means to calculate all flow details by using a minimum of model assumptions. The first approach often fails because the model assumptions applied are invalid. The second approach is often inapplicable to flows seen in reality because of unaffordable computational costs of such detailed calculations. Ways to overcome the problems of mainstream approaches are under investigation for decades. The usual approach is to combine both types of PDE. Because of the very different nature of equation types, the disadvantage of this approach is the uncontrolled appearance of both sorts of solutions of equations. The practical consequence is the current inability to perform reliable predictions of many realistic turbulent flow scenarios.

Based on previous theoretical work [1-10], our project provided a solution for the core problem of existing problems: the mathematically correct design of PDE which include both model and flow resolution components [11-45]. The novelty of this approach is the assessment of flow resolution implied by mathematics and set up of the communication and response mechanism of the components involved. Applications to challenging flow configurations provide convincing support for the functioning of simulation methods designed in this way. In addition, we solved another relevant mathematical problem arising from the need to involve adjustable model parameters in simulation methods, which is a known source of significant inaccuracy. Implied by exact mathematics we designed a dynamic calculation of such model parameters on the fly (i.e. during the simulation), which enables an optimal adjustment of simulation methods to conditions considered. Flow applications reveal the benefits of this dynamic modeling approach.

The practical advantage of our mathematics-based developments is the reliable capability to predict turbulent flow regimes that cannot be studied in terms of other analysis methods because corresponding applicability conditions cannot be satisfied. We informed the community about this capability to advance our understanding of the structure of turbulent flows and to provide guidelines for the evaluation of turbulence models [26-45]. We implemented our novel computational methods in OpenFOAM, a simulation code which is available to the public. The same applies to the implementation of our dynamic model. A relevant fact in this regard is also that the novel simulation methods can be relatively easily implemented by modifications of a variety of currently applied methods for the simulation of turbulent flows. The corresponding requirements are described in a several journal papers and specifically brought to the attention of several research groups by a variety of international conference presentations and research talks.

The solution approach applied is presented so far in regard to PDE usually applied to address relatively small-scale engineering problems. However, there is technically no difficulty to use the same approach to overcome corresponding problems that appear in other areas. This concerns, for example, the combination of very large-scale atmospheric boundary layer (ABL) simulations (which focus on the modeling of processes) with relative small-scale ABL simulations (which focus on the resolution of processes, for example the details of flow around wind turbines). This means, our approach provides the basis for a solution of the Terra Incognita problem raised by Wyngaard, an atmospheric science problem that is unsolved over decades.

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