5 Propulsion

5.1 PDF/FDF-Methods for the Prediction of Supersonic Turbulent Combustion

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Abstract

The paper presents an overview of recent developments of PDF/FDF-methods for the prediction of turbulent combustion with special emphasis on work performed within the SFB 255/TP A2. On the basis of a discussion of available methods for the calculation of turbulent reacting flows, significant conceptual shortcomings of currently applied PDF/FDF-methods are highlighted and new concepts to overcome these problems are described. A main purpose of developing PDF/FDF-methods is to use them for the solution of real technological turbulent combustion problems. However, this goal could not be achieved until now due to the unavailability of appropriate codes. It is reported in which way this fundamental problem has to and will be solved in the near future.

5.1.1 Introduction

The reliable prediction of turbulent combustion processes represents a need for many technological developments (for example, to enhance the performance of airbreathing propulsion systems). Measurements can be used for that only in a limited way because they are often laborious and prone to errors. In addition to that, they usually provide only an incomplete picture of the underlying physics.

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Hence, predictions of turbulent combustion processes have to be based, essentially, on results of numerical methods. In particular, one needs numerical methods which are available for prospective customers, applicable to a broad range of combustion problems, efficient and accurate.

From a conceptual point of view, so-called hybrid PDF-methods offer at this stage of development the most promising way to find an appropriate solution to that problem because they combine two important advantages: they are able to take nonlinear chemical conversion processes exactly into account, whereas they are (in contrast to other methods) realizable from a computational point of view, this means they can be used for technological applications. However, these methods apply different approximations for turbulent transport processes, which determine, therefore, the accuracy of predictions. With regard to this, currently applied hybrid PDF-methods are faced with significant problems: they are inconsistent by the use of different models for the same physical process, simulate turbulent mixing partly in contradiction to the observed physics and describe the turbulent transport of energy on the basis of not well-founded, empirical assumptions. Apart from that, one has to see that there are (in contrast to the main purpose of developing such methods) no applications of PDF-methods to the solution of real technological turbulent combustion problems at present: such codes only exist within a few groups of specialists at universities, and there is hardly a way for other people (potential industrial users) to make use of them. Accordingly, the work performed within the SFB 255/TP A2 was devoted to two basic tasks: to develop new solutions to overcome conceptual problems of hybrid PDF/FDF-methods, and to push the use of corresponding numerical methods for predictions of technological turbulent combustion processes. The realization of these tasks will be described here.

The paper contains an overview of basic methods for turbulent reacting flow calculations in Section 5.1.2. This results in the conclusion that hybrid PDF/FDF-methods are particularly appropriate for turbulent reacting flow calculations at this stage of development. Section 5.1.3 deals with a discussion of shortcomings of currently applied PDF/FDF-methods. Solutions that were developed to overcome these problems are described in Section 5.1.4. The development of corresponding numerical methods for turbulent combustion prognoses is the concern of Section 5.1.5. Finally, Section 5.1.6 deals with a discussion of future developments that can be expected in this field.

5.1.2 Methods for Turbulent Reacting Flow Calculations

The problem related to the calculation of turbulent reacting flows is that (due to the memory capabilities and speed of high-performance computers) there are clear limitations for applying direct numerical simulation (DNS), i.e., the direct numerical integration of the basic equations of fluid and thermodynamics. At present and in the foreseeable future one can only use DNS for the calculation of flows with relatively small Reynolds numbers (flows with a weak or

moderate turbulence intensity) [1–6]. However, flows of technological and environmental relevance are usually characterized by high Reynolds numbers, and the consideration of Reynolds number effects is clearly a requirement with regard to reacting flow simulations [7].

5.1.2.1 Basic Methods

Table 5.1.1 describes in an idealized way basic methods that are available for the calculation of turbulent flows with high Reynolds numbers. PDF- and FDF-methods [1–20] generalize RANS- and LES-methods, respectively: they do not only describe the dynamics of filtered variables, but they also provide the dynamics of fluctuations around these variables. The significant advantage of the extension of the set of variables considered is that nonlinear chemical reaction rates are given in a closed form, so that the consideration of chemical reactions does not require modelling [1]. Therefore, the use of PDF- and FDF-methods offers the best chance to develop numerical methods which are applicable to a wide range of flows and capable of providing reliable predictions of flows with complex chemistry.

The difference between PDF- and FDF-methods is given by the fact that PDF-methods represent a special case of FDF-methods. FDF-methods provide deviations from spatially-filtered values, where a suitable value for the filter width has to be chosen. In contrast to that, one works with a large filter width in PDF-methods, which is of the order of the correlation lengths of variables considered [2]. In general, FDF-methods are applied so that modelling assumptions do only concern the treatment of small-scale turbulent motions, this means one applies a small filter width [15–18]. Thus, one may expect that such predictions are more accurate than those of PDF-methods. However, the problem of the FDF approach consists in its computational costs, which do not allow the use of these methods for technical applications at present.

Table 5.1.1: Basic methods for the calculation of turbulent reacting flows with significant turbulence: large-eddy simulation (LES), Reynolds-averaged Navier-Stokes (RANS), filter density function (FDF) and probability density function (PDF) methods.

Equations for filtered variables	Hybrid methods	Equations for stochastic variables
LES-methods	Velocities: LES-methods Scalars: FDF-methods	FDF-methods
RANS-methods	Velocities: RANS-methods Scalars: PDF-methods	PDF-methods

5.1.2.2 Hybrid PDF/FDF-Methods

The stochastic treatment of both velocities and scalars (mass fractions of species and temperature) in full PDF/FDF-methods is expensive and often the reason for significant problems, in particular with regard to the calculation of compressible flows. Some details of stochastic velocity equations for such flows are still unclear at present, and the numerical solution of such equations is very complicated [2]. An alternative is given by the use of hybrid PDF/FDF-methods. Within that approach, one calculates velocity fields by RANS/LES-methods so that stochastic simulations can be restricted to the treatment of scalar transport, see Fig. 5.1.1 for an illustration. The advantage of this approach is that chemical reactions still can be handled formally exactly, and one does not have a significant problem with regard to the calculation of velocity fields: well-developed methodologies can be used for that. However, the full utilization of the capabilities of such hybrid methods requires their consistent and physically correct formulation. Unfortunately, this constraint is not satisfied for the currently applied methods, as shown in Section 5.1.3. This may be the cause for significant errors and hampers remarkably the assessment of differences between full and hybrid PDF/FDF-methods.

5.1.3 Some Deficiencies of Existing Hybrid PDF-Methods

Some deficiencies of existing hybrid methods will be pointed out in this Section. For simplicity, this discussion will be limited to the consideration of hybrid PDF-methods because FDF-methods are faced with the same problems.

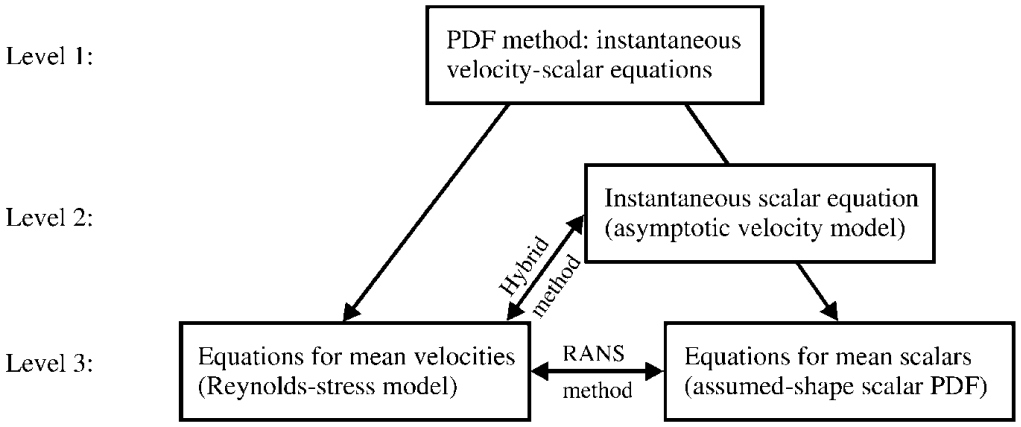

Figure 5.1.1: An illustration of the reduction of PDF-methods (stochastic equations for velocities and scalars) to RANS- and hybrid methods. Equations for mean velocities must be closed by algebraic or transport equation models for Reynolds stresses. The direct use of equations for mean scalars requires (assumed-shape) models for scalar PDFs to close mean reaction rates. A hybrid method is given by combining RANS equations for velocities with stochastic equations for scalars. The latter involve an asymptotic velocity model to calculate the transport of scalars in physical space. In consistency with that, one has to use an algebraic Reynolds stress model in RANS equations for velocities.

5.1.3.1 The Transport Problem

From the view point of a full PDF-method for velocities and scalars, the use of a hybrid method represents a simplification where (instead of equations for instantaneous velocities) deterministic velocity equations are solved, see Fig. 5.1.1. This reduction corresponds to the assumption that velocities are considered in a somewhat coarser scale where details of the acceleration statistics are not resolved [2].

The treatment of velocities and scalars on the basis of differently structured models then leads to constraints with respect to the consistency of these models, for example, due to the fact that both models require knowledge about velocity correlations (to close the turbulent transport term in equations for the averaged velocity field and the diffusion coefficient in stochastic equations for the transport of scalars in physical space). The application of a consistent hybrid method (see level 2 in Fig. 5.1.1) then requires the use of the same model for the Reynolds stress tensor in deterministic equations for velocities and stochastic equations for scalars. However, the latter is not guaranteed in existing methods. Anisotropy effects are considered in velocity equations whereas one applies isotropic diffusion coefficients in scalar equations. It is known that such imbalances may cause significant errors.

5.1.3.2 The Mixing Problem

As pointed out above, the significant advantage of using PDF-methods is the exact treatment of chemical reactions. This allows the reduction of the complex turbulence-chemistry interaction to the problem of describing the turbulent mixing of scalars appropriately. An example for such a mixing model represents the so-called "interaction by exchange with the mean" (IEM) model, which is (due to its simplicity) used in most of the PDF simulations of turbulent reacting flows. Within the frame of this model, a realization of the probability density function (PDF) of mass fractions m_α ($\alpha = 1, N$; N is the number of species considered) changes according to

$$\frac{d}{dt} m_\alpha = -\frac{C_\varphi}{2\tau}(m_\alpha - \overline{m}_\alpha) + S_\alpha \,. \tag{1}$$

The overbar refers to a mass density-weighted ensemble mean (a Favre mean), and S_α represents a chemical reaction rate, which follows from the reaction scheme considered. The mixing frequency $C_\varphi/(2\tau)$ is proportional to the mean frequency τ^{-1}, which characterizes the inertial range of turbulence. C_φ is a constant with a standard value $C_\varphi = 2.0$ [1].

However, the use of the model (1) is related to different problems. One problem is that this model is unable to simulate the initial stage of complex mixing processes in detail, which may be important with respect to accurate predictions of non-premixed combustion. Another problem is that this model is

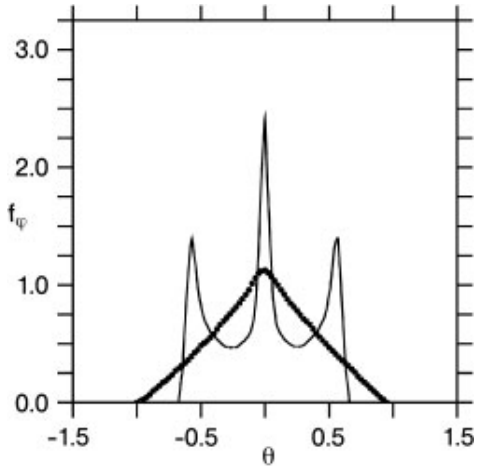

Figure 5.1.2: The dots present DNS data of the PDF $f_\varphi(\theta)$ of a passive scalar (for $\Phi_T = 0.6$, see Fig. 5.1.3). The thin solid line shows the corresponding predictions of the IEM-model for the evolution of the scalar PDF at this stage.

unable in general to calculate the correct asymptotic shape of scalar PDFs. It was shown by DNS data that scalar PDFs evolve (independent of the initial PDF shape) towards a Gaussian PDF in statistically homogeneous isotropic and stationary turbulence [21]. Instead, the PDF calculated by the IEM-model depends on the initial distribution also for very long times, see the illustration in Fig. 5.1.2. Such predictions are in contrast to the physics of turbulent mixing, such that significant errors in turbulent combustion simulations may occur.

5.1.3.3 The Energy Problem

The stochastic treatment of reacting scalar transport requires the incorporation of temperature in order to calculate instantaneous reaction rates in equations for species mass fractions. Furthermore, the consideration of energy variables (internal energy, enthalpy or temperature) is a requirement to calculate the pressure in compressible flows.

There exists a variety of ways to formulate equations for the transport of energy in compressible reacting flows: one can consider equations for the internal energy, enthalpy or temperature [22–24]. The way to be preferred currently consists in a combination of the stochastic model (1) with an equation for the enthalpy h. This equation can be written

$$\frac{d}{dt}h = -\frac{C_\varphi}{2\tau}\left(h - \overline{h}\right) + S_h\,. \tag{2}$$

S_h is a source term which follows from the basic equations of fluid and thermodynamics as

$$S_h = 2\nu S_{jk}^d S_{kj}^d + \frac{1}{\rho}\frac{dp}{dt} \,. \tag{3}$$

ν represents the kinematic viscosity, S_{jk}^d is the deviatoric part of the rate-of-strain tensor, p the pressure and ρ the mass density. S_{jk}^d, p and ρ are instantaneous (fluctuating) variables, and the problem is given by the fact that they appear as unknowns within the frame of combined mass fraction-enthalpy equations. Therefore, the model (2) is unclosed. To overcome this problem, one currently applies the following approximation [24]

$$S_h^{cl} = \overline{\frac{1}{\rho}\frac{dp}{dt}} \approx \frac{1}{\langle\rho\rangle}\left\langle\frac{\partial p}{\partial t} + U_k\frac{\partial p}{\partial x_k}\right\rangle = \frac{1}{\langle\rho\rangle}\left[\frac{\partial\langle p\rangle}{\partial t} + \langle\bar{U}_k\rangle\frac{\partial\langle p\rangle}{\partial x_k} + \left\langle u_k\frac{\partial p}{\partial x_k}\right\rangle\right]; \tag{4}$$

this means one neglects the first term on the right-hand side of relation (3), which simulates an increase of internal energy due to viscous friction, and replaces the pressure change by the corresponding mean value (the symbol $\langle\ \rangle$ refers to an ensemble mean). The resulting problem to explain the last term of relation (4) by means of variables that are known is then solved on the basis of usual parametrizations, which are applied within the framework of turbulence modelling [24].

This previously applied way to involve energy variables in PDF-methods for reacting flows is characterized by different disadvantages. First, one neglects fluctuations of dp/dt, which may be important in supersonic flow simulations [22]. This approximation also disagrees with the goal of PDF-methods to describe the dynamics of scalar fluctuations in a consistent way (the neglect of fluctuations of dp/dt corresponds to the neglect of temperature fluctuations, which appear, for instance, in instantaneous reaction rates). Furthermore, one produces additional closure problems in this way (see the last term in (4)), which cannot be solved appropriately for inhomogeneous reacting flows. The closure (4) also has disadvantages from a numerical point of view, because the calculation of the substantial derivative of the mean pressure causes several problems [25]. Due to these reasons, the currently applied way to involve energy variables in hybrid PDF-methods cannot be seen as a satisfactory solution to this problem and requires methodological improvements.

5.1.4 New Theoretical Concepts

The work performed within the SFB 255/TP A2 to overcome the conceptual problems of hybrid PDF-methods pointed out in Section 5.1.3 will be described next.

5.1.4.1 The Transport Problem

A solution to the transport problem pointed out above may be obtained by reducing the underlying transport equation for the joint velocity-scalar PDF, see Fig. 5.1.1. In this way, one obtains a generalization of methods applied usually, where contributions due to anisotropy and shear (which appear in general in anisotropic and inhomogeneous flows) are involved in calculations of scalar transport in physical space [2]. The result is a consistent hybrid method where the same sub-models are used in equations for velocities and scalars.

5.1.4.2 The Mixing Problem

An important goal of the work performed within the SFB 255/TP A2 was the improvement of models for the turbulent mixing of scalars. Such an improvement of the IEM-model can be obtained on the basis of the so-called projection operator technique [2, 19, 26]. In this way, the IEM-model can be generalized (in a non-expensive way) so that complex mixing processes within the initial stage of turbulent mixing and the evolution of scalar PDFs can be simulated correctly in agreement with the boundedness constraint [1].

Some essential properties of this new model are described in the following by considering a binary mixing process. The test case considered is given by DNS data of Juneja and Pope [21] who investigated the mixing of passive scalars in statistically stationary isotropic and homogeneous turbulence. Results obtained by means of the new mixing model and a simplified version are given in Fig. 5.1.3 in comparison to the corresponding DNS data. This comparison demonstrates the good performance of the new mixing model. The evolution of the PDF in the initial and asymptotic stages of mixing is described in agreement with DNS data. Figures 5.1.3/c1 and 5.1.3/c2 show that the simplified version of the new mixing model also enables good predictions, if a very detailed resolution of complicated mixing processes in the initial stage of turbulent mixing is not essential. The simplified mixing model represents, basically, a stochastic variant of the IEM-model. Thus, it is easy to use and does hardly require more computational effort than the IEM-model.

5.1.4.3 The Energy Problem

As pointed out above, the presently applied formulation of the stochastic energy transport on the basis of an enthalpy equation is related to the appearance of a variety of problems. A way to overcome these problems has been developed as part of the work performed within the SFB 255/TP A2. The basic idea is to use a temperature formulation of the energy transport (a temperature equation) in conjunction with the consistent use of approximations that are a

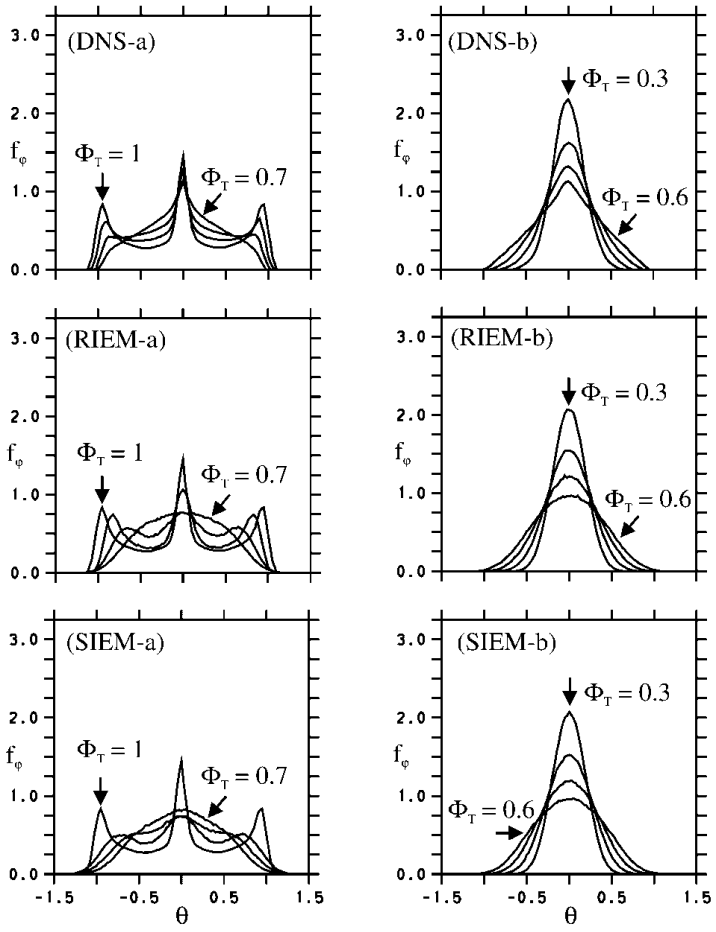

Figure 5.1.3: The evolution of a scalar PDF $f_\varphi(\theta)$ as function of the progress variable Φ_T, which runs from unity to zero in time. The DNS data of Juneja and Pope [21] are given in (DNS-a) and (DNS-b), and the corresponding prognoses of the new mixing model in (RIEM-a) and (RIEM-b). The Figs. (SIEM-a) and (SIEM-b) show the performance of a simplified version of the new model.

consequence of reducing a transport equation for the joint PDF of velocities and scalars to a hybrid method, see Fig. 5.1.1 for an illustration. The simplicity and consistency of this concept offers significant advantages compared to the use of other methods [2].

5.1.5 The Use of PDF Combustion Codes

5.1.5.1 The Current Use of PDF/FDF-Methods

One goal related to the development of PDF/FDF-methods is to investigate the physics of turbulent flows with high Reynolds, Schmidt and Damköhler num-

bers, which cannot be assessed by means of DNS. FDF-methods are particularly appropriate for that due to the fact that large-scale motions are treated without modelling. However, their application to such studies is still in an initial stage. In contrast to that, PDF-methods, which represent a special case of FDF-methods (see Section 5.1.2.1), are already applicable to such investigations. Due to their close relationship to FDF-methods, such PDF-studies are well appropriate to prepare the use of corresponding FDF-methods.

Another very important goal of the development of PDF/FDF-methods is their use for predictions of technological combustion processes. Due to their lower computational costs, PDF-methods are more appropriate for that than FDF-methods. However, despite of important conceptual advantages of PDF-methods compared to RANS-methods, one has to see that these methods are not used at present for the solution of technological problems. The main reason for that is given by their requirements with regard to the memory and speed of computers, which are significantly higher than those related to the use of conventional turbulence models, which are based on deterministic equations. The constraints for the application of PDF-methods can be illustrated in the following way. The calculation of industrial flows often requires a number of grid points which is of the order of 100^3. Without adopting techniques for the reduction of statistical errors, one needs 10^4 or 10^6 realizations (particles) in each box of the grid to keep the statistical error of calculations below 1% or 0.1%, respectively. The treatment of the evolution of at least 10^{10} particles then represents a huge challenge. To handle this problem, the use of PDF-methods for the solution of technological problems requires the careful analysis of strategies for the numerical solution of stochastic equations and error sources in conjunction with the development of efficient techniques for the reduction of errors in order to limit the significant computational costs and memory requirements. These questions were solved recently only within a few research groups of specialists at universities who apply PDF codes for scientific investigations. Such codes, which are the result of long-standing investments, are not made accessible to potential industrial users in general, which would be related to an enormous effort to explain the use of codes and treatment of problems. Apart from that, one finds that such developments often do not correspond to technological requirements with regard to computing facilities, generation of grids and turbulence modelling.

This lack of convincing technological applications of PDF-methods has several negative consequences. Due to the fact that the development of FDF-methods is still in its initial stage, it supports doubts regarding the advantage of developing such methods, which results in an unsatisfactory status of research funding in this area so that these developments are carried out by only a few groups in the world. This leads to the problem to keep pace with modern developments, for instance with regard to the generation of unstructured grids, the improvement of turbulence models and the tabulation of chemistry. Another problem is the implied limited number of applications (the availability of PDF codes for many users at universities and the industry offers the chance to obtain a much broader knowledge regarding the advantages and disadvantages of different models).

5.1.5.2 New Developments

Fortunately, there are promising new activities of Fluent Inc. (a software company that offers a computational fluid dynamics (CFD) code called FLUENT) to overcome the problem related to the lack of using PDF-methods for technological applications. Fluent Inc. was the first firm that involved PDFs in combustion simulations. That concerned an advanced RANS-method where the shape of the scalar PDF has to be provided to close mean reaction rates in equations for the scalar transport (so-called assumed-shape PDF-models, see the illustration in Fig. 5.1.1). However, one has to see that such assumed-shape PDF-methods do not contain the required physics in order to simulate the processes that take place in non-premixed flames [2]. Therefore, one finds significant shortcomings of assumed-shape PDF-models [27], for example, concerning the calculation of non-premixed hydrogen and methane turbulent jet flames [28, 29].

The natural next step was the integration of a hybrid PDF-method (see Fig. 5.1.1) in FLUENT [30], which was performed in the beginning of 2003 in collaboration with Professor S.B. Pope (Cornell University). It is worth noting that the incorporation of ISAT-routines [31] in FLUENT offers the possibility to calculate then complex chemical reactions in a very efficient manner. Nevertheless, it has to be pointed out that this code still does not offer solutions for the transport, mixing and energy problems described above. Furthermore, the code is not tested until now with regard to its numerical accuracy and performance as combustion model.

5.1.5.3 Common Activities to Develop a New Combustion Code

Obviously, the new development of Fluent Inc. offers the great chance to overcome the problems pointed out at the end of Section 5.1.5.1. In particular, it turned out that the activities at the Technical University of Munich (Fachgebiet Strömungsmechanik) and Fluent Inc. can be combined in a way that this development is made much more efficient. The basic configuration of this collaboration is that Fluent Inc. implements the methodological improvements reported in Section 5.1.4 in the PDF code of FLUENT. This allows to test the relevance and performance of the new models on the basis of various applications. From the view point of Fluent Inc., this work is also very helpful because it supports the code testing and provides evidence for the performance of the new PDF code as a combustion model. Accordingly, the Technical University of Munich (Fachgebiet Strömungsmechanik) and Fluent Inc. Germany agreed in corresponding common activities to test and improve the existing FLUENT code.

In a first application, the PDF code of Fluent Inc. is used to simulate a three-dimensional supersonic turbulent channel flow where a passive scalar is injected from one wall and removed from the other wall. For that flow, DNS data are available (at different Reynolds and Mach numbers) for comparisons, which were produced within a project funded by the DFG [32, 33]. One goal of

these comparisons between PDF simulations with DNS data is to validate the numerical accuracy of the PDF code. Due to its simplicity, the flow considered is very appropriate for that. A relevant question concerns, for instance, the number of particles that are required within the Monte Carlo simulation, which is essential to the efficiency of calculations. From a scientific point of view these comparisons are also relevant since they can be used for the assessment of the relevance of the methodological improvements described in Section 5.1.4. It is, for example, possible to get a better insight into the relevance of anisotropy on the transport of scalars in physical space and the influence of compressibility on the PDF of scalars. Further, the DNS data are very appropriate to investigate the advantages of the temperature formulation of the energy transport in comparison to the enthalpy formulation applied previously. The results of these investigations are currently prepared for publication. It is also worth noting that such studies of the dynamics of scalars are also relevant to the construction of FDF equations, where very similar questions have to be answered [2].

Further applications of this PDF code are planned in the years 2005–2006. This concerns the simulation of different subsonic and supersonic flames. These investigations will be done to reach two goals. The first one is to assess the performance of the improved PDF code (the relevance of methodological improvements). The second one is to provide evidence for the applicability and numerical accuracy of the PDF combustion code developed by Fluent Inc.

5.1.6 Prospects for Further Developments

Questions related to the further development of computational methods will be addressed now. This concerns, for example, the question whether (in the light of the development of FDF-methods) the further development of PDF-methods still will be in agreement with the general trend of development that can be expected. This is the case, as will be shown in Section 5.1.6.1. This results then in the question about the future relation of PDF- and FDF-methods, which will be discussed in Section 5.1.6.2. More detailed explanations of these questions may be found elsewhere [2].

5.1.6.1 The Current and Future Use of Computational Methods

To understand needs for further methodological developments, let us have a closer look at current and future applications of turbulence models for solving industrial (technological and environmental) problems.

The current use of computational methods is illustrated in Fig. 5.1.4, which closely follows an analysis of Pope [3]. Obviously, most applications are performed on the basis of the simplest (RANS) models. In particular, the k-ε model

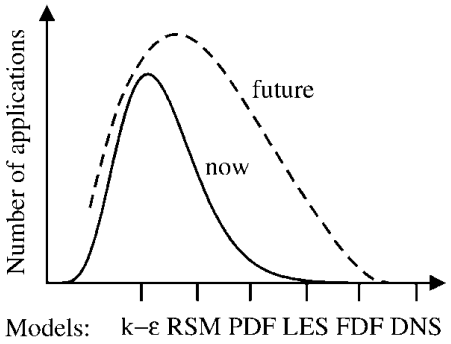

Figure 5.1.4: A slightly extended prognosis of Pope [3], who considered the current and future (next 10–20 years) use of turbulence models for industrial applications. RANS-models are split into algebraic (k-ε) and Reynolds stress models (RSM) which apply transport equations to calculate Reynolds stresses.

and (less often) Reynolds-stress models (RSM), which make use of transport equations to obtain the Reynolds stresses, are the models which are usually employed. As pointed out above, the use of RANS equations for reacting flow calculations is faced with serious problems regarding the closure of mean reaction rate terms. This problem can be solved by extending RANS to PDF-methods.

Regarding the future use of computational methods for industrial applications, an important conclusion from previous developments is that a simple displacement of the distribution in Fig. 5.1.4 towards more advanced methods (an abandonment of RANS in favour of LES) will probably not take place. The general trend is to use the simplest models as much as possible. Thus, it is likely that the contributions of PDF-, LES- and possibly FDF-methods grow, but most of the applications will be still performed on the basis of RANS(PDF)-methods.

5.1.6.2 Some Challenges

A significant problem related to the use of RANS-models for industrial applications is the assessment and optimization of the model performance. In general, the use of DNS is too expensive for that purpose. Measurements are also very expensive and provide only partial (and often relatively inaccurate) information. With regard to this, it is essential to note that possible deficiencies of RANS-models are closely related to the underlying relative coarse filtering [2]. The most natural way to assess the suitability of RANS-models, therefore, is to investigate the consequences of applying (by the use of a RANS-method) a large filter width Δ. This can be done by adopting LES predictions for comparisons, provided the LES equations recover the RANS-model considered in the large-Δ limit. The latter is not guaranteed, however, in the majority of existing LES-models. This leads to the need for the construction of unified models that can be used, depending on the resolution, as either LES- or RANS-methods, see

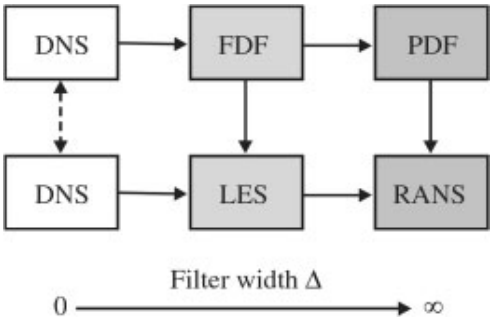

Figure 5.1.5: An illustration of unified turbulence models. Equations for filtered variables are given below. They can be applied as DNS-, LES- or RANS-models, depending on the choice of Δ. The upper line shows corresponding stochastic models (SGS fluctuations vanish for $\Delta \to 0$ such that FDF-methods reduce to DNS).

Fig. 5.1.5 for an illustration and [34]. It is relevant to note that the requirement of developing unified turbulence models is not restricted to the need to represent RANS and PDF equations as large-Δ limits of LES and FDF equations, respectively, but a corresponding deepening of the relations between DNS and LES/FDF equations will also be very helpful [2].

On the other hand, it is obvious that the computational realization of unified turbulence models requires solutions for several new questions. Instead, the advantage of such efforts may be enormous: it offers the chance for a significant reduction of expensive experiments, which are still needed to assess the performance of numerical prognosis methods.

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