Homework 1 (MATH 5490-01) **Name (Print):**
Due date: Monday, Sept. 28, 2009

Consider the beta PDF defined by

$$f(x) = \begin{cases} \dfrac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

Here, B(a, b) is the so called beta function that is defined by

$$B(a,b) = \int_0^1 y^{a-1}(1-y)^{b-1} dy.$$

The beta function can be calculated via its relation to the gamma function,

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$

For $0 \le x \le 1$, a polynomial approximation for the gamma function is given by

$$\Gamma(x+1) = 1 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5,$$

where

$$a_1 = -0.5748646, \quad a_2 = 0.9512363, \quad a_3 = -0.6998588$$
$$a_4 = 0.4245549, \quad a_5 = -0.1010678.$$

1. Derive a formula for the moments of the beta PDF of any order.

2. Use this formula to determine the mean and variance of this PDF in dependence on the model parameters a and b.

3. Determine the model parameters a and b as functions of the mean value and variance.

4. Assume that the mean value is equal to 1/2. Plot the beta PDF for the three standard deviation values 0.4, $0.5/3^{1/2}$, and 0.15.

5. Provide examples for the use of the three beta functions shown in response to question 4.