Homework 4 (MATH 5490-01) **Name (Print):**
Due date: Monday, Nov. 9, 2009

Consider the following model for the population P_n:

$$\frac{P_n - P_{n-1}}{\Delta t} = P_{n-1}(1 - P_{n-1})\left(\mu + \sigma \frac{\Delta W_n}{\Delta t}\right). \quad (1)$$

Here, $\Delta W_n / \Delta t$ is the discrete derivative of a Wiener process. The model parameters are given by $P_0 = 0.25$, $\Delta t = 0.01$, $\mu = 1$, and $\sigma = 2$.

1. Analysis of moment equations:
   a) Plot the mean and standard deviation of P_n that are determined by the numerical solution of equation (1) as function of n.
   b) Consider the equation system for the moments of first and second order of P_n which is implied by equation (1). Use the beta PDF to parametrize the third-order and fourth-order moments in this equation system in terms of first-order and second-order moments. Plot the mean and standard deviation of P_n that are determined by the equations for first-order and second-order moments (the beta function is used to close unknown terms) as function of n.
   c) Discuss the suitability of using the beta PDF to close moment equations by comparing the functions calculated in 1a) and 1b).

2. Analysis of time to extinction statistics:
   a) The time to extinction is defined as the earliest time at which a realization P_n is found in the interval $0 \le P_n \le 10^{-8}$. Solve the equation (1) up to t = 50. Determine the time to extinction of all realizations that approach the equilibrium value zero.
   b) Calculate and discuss the meaning of the time to extinction PDF.
   c) Calculate the mean time to extinction for the cases $P_0 = 0.25$, $P_0 = 0.5$, and $P_0 = 0.75$. Discuss the influence of P_0 on the mean time to extinction.