Homework 5 (MATH 5490-01) **Name (Print):**
Due date: Monday, Nov. 30, 2009

1. Consider the following model for the PDF f(v, t) of molecular velocities v in one direction:

$$\frac{\partial f(v,t)}{\partial t} = \frac{\partial}{\partial v}\left[\frac{v-V}{\tau_m} f(v,t)\right] + \frac{2e}{3\tau_m}\frac{\partial^2 f(v,t)}{\partial v^2}. \quad (1)$$

Here, τ_m is the constant characteristic time scale of molecular velocity fluctuations, and V and e are constant model parameters.

a) Calculate the stationary PDF f(v) (which is independent of time t) by integration of equation (1).
Hint: The first integration constant can be determined by the constraint that both f(v) and $\partial f(v)/\partial v$ have to be zero if $v \to \infty$. The second integration constant can be determined by the normalization constraint for f(v).

b) Explain the meaning of V and e by relating these variables to the mean and variance of f(v). Explain why the stationary PDF f(v) does not depend on τ_m.

2. The development of a certain population in time t is described by the following equation for the population PDF f(p, t):

$$\frac{\partial f(p,t)}{\partial t} = \frac{\partial}{\partial p}\left[\frac{p-C}{\tau} f(p,t)\right] + \frac{\partial^2 D f(p,t)}{\partial p^2}. \quad (2)$$

Here, τ is the constant characteristic time scale of population fluctuations, and C and D are constant model parameters.

a) Use equation (2) to find the mean <P> and variance $<\tilde{P}^2>$ as functions of t.

b) Find the conditional PDF f(p, t | p', 0) such that its parameters α and β are given as explicit functions of t.
Hint: The best way to address this question is to consider the equations for $\alpha - <P>$ and $\beta - <\tilde{P}^2>$.

c) Write the PDF f(p, t) in dependence on the conditional PDF f(p, t | p', 0) and any initial PDF f(p', 0). What is the PDF f(p, t) if $t \to \infty$?

d) Calculate the asymptotic ($t \to \infty$) PDF f(p, t) for the case that $D \to 0$. Explain the meaning of your result.