Statistical modeling of compressible turbulent channel flow

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Several questions that are relevant to turbulence modeling are addressed on the basis of recently obtained direct numerical simulation results of turbulent supersonic channel flow. In particular, this concerns the turbulence frequency production mechanism, wall damping effects on turbulence model parameters, and the relevance of compressibility effects. Limited support is found for usually applied models for the turbulence frequency production and wall damping effects. In contrast to that it is shown that turbulence frequency production mechanisms and wall damping effects may be explained very well on the basis of a frequency scaling that characterizes mean flow changes. The influence of compressibility is found to be relevant.

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1 Introduction

Most of the simulations of turbulent reacting flows are performed within the frame of Reynolds-averaged Navier-Stokes (RANS) methods or more general probability density function (PDF) methods (their application allows to describe some important processes exactly, as, for example, chemical conversion processes) [1]- [3]. The reason for that is given by the fact that the computational costs of RANS and PDF methods are much lower than those of corresponding large eddy simulation (LES) and filter density function (FDF) methods. However, their relative simplicity is also the reason for some significant problems, which limit the accuracy of RANS and PDF methods. These problems will be addressed by taking reference to recently obtained direct numerical simulation (DNS) results of turbulent supersonic channel flow [4]- [5].

2 The turbulence frequency production

A first problem is given by the fact that there are good concepts available to model the evolution of velocity and scalar fields by stochastic or simpler deterministic methods, but to close such equations one has to provide the turbulence frequency ω (or dissipation rate $\epsilon = k\omega$ of turbulent kinetic energy k) which determines the characteristic time scale $\tau = 1/\omega$ of turbulent motions. Unfortunately, the basis for constructing equations for ω or ϵ is weak because the most important terms in these equations, the standardized source rates S_ω and S_ϵ, are unknown. Kolmogorov's notion was that ω is associated with the smallest scales of turbulence, and thus has no direct interaction with the mean motion. He concluded that S_ω should be independent of the production of turbulence and approximated by a constant [6]. Nevertheless, in most applications S_ω is considered as a linear function of the production-to-dissipation ratio P/ϵ of k [7]. However, the general validity of this assumption is questionable. To get further insight into this problem, this question was addressed (with regard to the source rate S_ϵ in the dissipation equation) by means of the renormalization group (RNG) theory [8]. However, Smith and Woodruff state: "Even though they may be motivated physically or otherwise, it is evident that many steps of the renormalization-group scale-removal procedure as currently formulated are mathematically not rigorously justified" [8]. Therefore, the question of how the source terms S_ω and S_ϵ scale with turbulence characteristics cannot be treated as being already clarified.

An analysis of this question on the basis of recently obtained DNS data of turbulent supersonic channel flow [5] reveals the following. Very limited support is available for assuming that S_ω is a linear function of P/ϵ: such a linear relation can only be found in the near wall region before P/ϵ reaches its maximum. With regard to S_ϵ, it is found that the standard model $1.92 - 1.44P/\epsilon$ can only indicate the general trend of S_ϵ variations (with a significant inaccuracy). In contrast to these findings, an accurate parametrization for S_ω can be derived on the basis of a new frequency scaling that characterizes mean flow changes. The mechanism of S_ω variations can be well explained in this way: S_ω evolves towards an equilibrium value. It is also found that other formulations of scale determining equations are less appropriate.

3 Wall damping effects

A second problem is related to the optimization of the performance of turbulence models. The efficiency of turbulence models mainly arises from the fact that turbulence model parameters (as C_μ) are introduced via the parametrization of correlations of turbulent velocities and scalars which appear as unknowns in turbulence models. Originally, such model parameters were assumed to be constant, but many investigations indicated significant shortcomings as a consequence of this assumption. This concerns, in particular, the modeling of wall-bounded flows, which are relevant to most applications. It turned out that the

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performance of turbulence models can be significantly improved by adopting varying turbulence model parameters, so that the damping effect of walls can be taken into account. Basically, two concepts were applied previously for the construction of such damping functions [1], [7]: the scaling of coefficients in terms of normalized wall distances, or their scaling with a turbulence Reynolds number Re_L. However, there are many questions regarding the validity and generality of these concepts.

The investigation of these questions based on turbulent supersonic channel flow DNS data [5] supports conclusions of Rodi and Mansour [9]: there is no support available for Re_L scalings. Scaling concepts based on wall distances do not represent an alternative. Such scaling concepts do not provide turbulence models which are invariant under the Galilean transformation, and they do not have a broad range of applicability: one needs different (inner and outer) scalings for different flow regions, and inner scaling turned out to be inapplicable to compressible flow. The use of such concepts may also become very problematic with regard to flows in complex geometries (flow along a right angled corner) or multi-component reacting flows. Thus, there is a need for the development of wall effect scalings that are independent of wall parameters. It is found that the use of a frequency scaling that characterizes mean flow changes offers significant advantages. The main reason for that is the reference to a local flow state that is independent of wall properties (the latter is very helpful, for example, with regard to simulations of flows in complex geometries or multi-component reacting flows). By adopting the new scaling, the mechanism of C_μ variations can be well explained: in correspondence to S_ω variations one observes a trend towards an equilibrium value of C_μ.

4 Compressibility effects

A third problem concerns the development of solutions for the two problems described above (or, more general, the development of turbulence models) for variable-density flows, which is relevant to turbulent combustion calculations. Compressibility effects that were observed in such flows may be differentiated, basically, into dilatational and structural compressibility effects. Dilatational compressibility effects were observed in homogeneous shear flows [10]: independent of the gradient Mach number one finds that the ratio of both the dilatational dissipation rate and pressure-dilatation correlation to the solenoidal dissipation rate is about 10%. However, the relevance of dilatational compressibility effects to wall-bounded flows seems to be very low. In contrast to that, structural compressibility effects (changes of the dimensionless anisotropy tensor due to a reduction of the turbulent kinetic energy redistribution) were found to have a very significant effect on the production and dissipation of turbulence in homogeneous shear flows [10], which requires corresponding modifications of turbulence models [11]. With regard to wall-bounded flows there is certainly the need for further investigations of their significance and of appropriate ways to incorporate these effects in turbulence models. It has to be clarified, for example, whether the parameters S_ω and C_μ are significantly affected by such structural changes, and whether the scaling of structural compressibility effects in terms of the gradient Mach number is an appropriate concept also for wall-bounded flows.

Turbulent supersonic channel flow DNS data [5] reveal that compressibility effects are relevant to the flows considered: the characteristic length and time scales of turbulent eddies and production and dissipation of turbulent kinetic energy are clearly affected by compressibility. Regarding the relevance of compressibility effects to turbulence model parameters one observes that such effects do not modify turbulence parametrizations in the mean velocity, temperature, mass fraction and turbulent kinetic energy equations. Hence, the evolution of C_μ is also unaffected: compressibility modifies C_μ only via the wall condition. However, compressibility effects are found to be relevant regarding the evolution of S_ω. This variation should be taken into account in turbulence models.

5 Conclusions

The conclusions obtained are the following ones. Limited support is found for standard models used in almost all predictions of turbulent flows of practical relevance [7]. The development of accurate turbulence models is possible, but such models depend on the flow considered. A very promising general methodology to develop such flow-dependent models is given by the application of unified turbulence models [12]. This approach allows the use of a few accurate (but expensive) FDF or LES simulations to determine, validate and optimize the performance of more efficient PDF or RANS turbulence models.

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